13.2 Vector Calculus

Giving Away the Punchline…

Do calculus component-wise!

$$\boldsymbol{r}'(t) = \langle x'(t), y'(t), z'(t) \rangle$$

$$\boldsymbol{r}''(t) = \langle x''(t), y''(t), z''(t) \rangle$$

Entry task Like HW 13.2 / 2–5

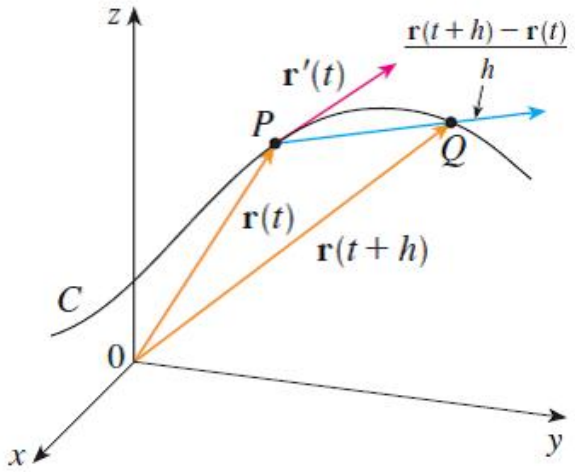
$$\boldsymbol{r}(t) = \langle t, \cos(2t), \sin(2t) \rangle$$

Find **r**′(t), **r**(π/4), and **r**′(π/4).

Interpreting the derivative vector HW 13.2 / 2–4

Let's take a look at this definition bit-by-bit…

$$\boldsymbol{r}'(t_0) = \lim_{h \to 0} \frac{\boldsymbol{r}(t_0 + h) - \boldsymbol{r}(t_0)}{h}$$



Unit tangent vectors HW 13.2 / 6–7

The unit tangent vector points in the direction of the tangent, but has length one..

$$\boldsymbol{T}(t) = \frac{\boldsymbol{r}'(t)}{|\boldsymbol{r}'(t)|}$$

Entry task continued

$$\boldsymbol{r}(t) = \langle t, \cos(2t), \sin(2t) \rangle$$

Find **T**(t), then evaluate **T**(π/4).

Cool and Important Side Fact:

T(t) and **T**'(t) are always perpendicular

Tangent lines HW 13.2 / 8

Step 1: Find the point $\mathbf{r}(t_0)$.

Step 2: Find a direction $\mathbf{r}'(t_0)$.

$$< x, y, z > = \boldsymbol{r}(t_0) + u\,\boldsymbol{r}'(t_0)$$

continued (again)

$$\boldsymbol{r}(t) = \langle t, \cos(2t), \sin(2t) \rangle$$

Find parametric equations for the tangent line at t = π/4.

Winter 2015 Exam 1 — Loveless

$$x = 2t,\ y = 5,\ z = t^2 - 10t$$

(a) There is one point on the curve at which the tangent line is parallel to the xy-plane.
Find the tangent line at this point.

Angles of intersection HW 13.2 / 9–10

(b) Find the angle of intersection of

$$x = 2t, y = 5, z = t^2 - 10t$$

and

$$x = 7 + u,\ y = 2u + 11, z = u^2 + u - 22.$$

Integrate component-wise HW 13.2 / 11

$$\int r'(t)dt = \langle \int x'(t)dt\,, \int y'(t)dt\,, \int z'(t)dt \rangle$$

Each component has its own constant. Use the initial position to find all three constants. We will discuss this more in 13.4.

Example

$$\boldsymbol{r}'(t) = \langle e^{3t}, t^4, t\sin(t) \rangle$$
$$\boldsymbol{r}(0) = \langle 1, 3, -2 \rangle$$

Find **r**(t).

Derivatives Quick Review

A basic example: Write down the derivative of

$$g(x) = x^3\cos(2x) + \sqrt{1 + e^{3x}}$$

Calculus Fact Sheet

Essential Derivative Rules

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$	
$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(b^x) = b^x \ln(b)$	
$\frac{d}{dx}(\sin(x)) = \cos(x)$	$\frac{d}{dx}(\tan(x)) = \sec^2(x)$	$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$
$\frac{d}{dx}(\cos(x)) = -\sin(x)$	$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$	$\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{x^2+1}$	$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{x\sqrt{x^2-1}}$
$(FS)' = FS' + F'S$	$\left(\frac{N}{D}\right)' = \frac{DN' - ND'}{D^2}$	$[f(g(x))]' = f'(g(x))g'(x)$

From my Calculus 1 Fact Sheet:
math.washington.edu/~aloveles/Math126Materials/CalculusFactSheet2.pdf

Antiderivatives Quick Review

A basic example: Find $f(t)$, if

$$f'(t) = 6\sqrt{t} + \sin(t) + 9te^{t^2}$$

with $f(0) = 7$.

From my Calculus 1 Fact Sheet:
math.washington.edu/~aloveles/Math126Materials/CalculusFactSheet2.pdf

Essential Integral Rules

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$	$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln\lvert ax+b\rvert + C$
$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$	$\int b^x\,dx = \frac{1}{\ln(b)}b^x + C$
$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C$	$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + C$
$\int \sec^2(x)\,dx = \tan(x) + C$	$\int \csc^2(x)\,dx = -\cot(x) + C$
$\int \sec(x)\tan(x)\,dx = \sec(x) + C$	$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$
$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\int \tan(x)\,dx = \ln\lvert\sec(x)\rvert + C$	$\int \cot(x)\,dx = \ln\lvert\sin(x)\rvert + C$
$\int \sec(x)\,dx = \ln\lvert\sec(x)+\tan(x)\rvert + C$	$\int \csc(x)\,dx = \ln\lvert\csc(x)-\cot(x)\rvert + C$

$$\int \sec^3(x)\,dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln\lvert\sec(x)+\tan(x)\rvert + C$$

For Ch. 13, you MUST remember:

- how to find +C.
- how to use **u-substitution**.
- How to use **integration by parts**.